Exercise 2: Probabilistic Foundation of the True Misclassification Rate ( 1.5 + 1.5 + 1 = 4 Points)

Consider a sample space Omega = {o{1}, o{2}, o{3}, o{4}, o{5}, o{6}} with six outcomes; i.e., each elementary event {o{i}} corresponds to observing one of six distinct objects. Let X subset R ^ 2 be a feature space, C = {0, 1} be of two classes, and P be a probability measure defined on {P}. Further, let X : Omega -> X and C : Omega -> C be two random variables defined according to this table:

A table with numbers and letters

Description automatically generated

1. Specify the joint distribution function p(x, c) := P (X=x, C=c) by completing this table:

|  |  |  |
| --- | --- | --- |
| X | C | Ρ(x,c) |
| (0,0)T | 0 | 0.1 |
| (0,1)T | 0 | 0.3 |
| (0,1)T | 1 | 0.3 |
| (1,0)T | 0 | 0.1 |
| (1,0)T | 1 | 0.2 |

1. Specify the Bayes classifier y\*() by completing this table (potentially more than one correct answer):

|  |  |  |
| --- | --- | --- |
| X | C | Ρ(x,c) |
| (0,0)T | 0 | 0.1 |
| (0,1)T | 0 | 0.3 |
| (1,0)T | 1 | 0.1 |

1. Specify the true misclassification rate Err ∗ of the Bayes classifier.

Err\* = p( X = (0,1)T, C = 0)